

Roots Under Particular Condⁿ

$$ax^2 + bx + c = 0$$

1) $b = 0$

$$ax^2 + c = 0$$

$$x = \pm \sqrt{\frac{c}{a}}$$

Roots opp sign

(2) $c = 0$

$$ax^2 + bx = 0$$

$$x(ax + b) = 0$$

$$x = 0, x = -\frac{b}{a}$$

One Root = 0

(3) $b = c = 0$

$$ax^2 = 0$$

$$x = 0, 0$$

Both Root = 0

(4) $a = c$

$$\alpha \cdot \beta = \frac{c}{a} = 1$$

$$\beta = \frac{1}{\alpha}$$

Reciprocal Roots

$$\alpha = -5, \beta = 2 \quad \left| \begin{array}{l} \alpha + \beta = -3 \\ \alpha \cdot \beta = -10 \end{array} \right.$$

$$|\alpha| = |-5|$$

$$= 5 > 2$$

$$|\alpha| > \beta$$

(5) a, b, c Same Sign

$$\alpha + \beta = -\frac{b}{a} = -ve$$

$$\alpha \cdot \beta = +ve$$

$$\alpha, \beta = -ve$$

(6) a, b, c alternative
opp sign.

$$a+ \quad b- \quad c+$$

$$\alpha + \beta = +ve$$

$$\alpha \cdot \beta = +ve$$

$$\alpha, \beta = +ve$$

(7) Sign of $a =$ Sign of b
 $\neq$ Sign of c

$$a+ \quad b+ \quad c-$$

$$\alpha + \beta = -ve$$

$$\alpha \cdot \beta = -ve$$

$$\alpha = -ve \text{ \& } |\alpha| > \beta$$

Q If $\alpha, \beta (\alpha < \beta)$ are Root of
 $x^2 + bx + c = 0$ ($c < 0 < b$)
 then P.T. $\alpha < 0 < \beta < |\alpha|$

$$x^2 + bx + c = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix} \quad \begin{matrix} b = +ve \\ c = -ve \end{matrix}$$

$$\alpha + \beta = -b = -ve$$

$$\alpha \cdot \beta = c = -ve$$

$$\begin{array}{l|l} \alpha = -5, \beta = +2 & |\alpha| > \beta \\ \hline \alpha + \beta = -3 & \alpha < \beta \\ \alpha \times \beta = -10 & \end{array}$$

Q If α & β are Root of $ax^2 + bx + c = 0$
 & $S_n = \alpha^n + \beta^n$ Find the value of

$$aS_n + bS_{n-1} + cS_{n-2} = ?$$

$$ax^2 + bx + c = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix} \Rightarrow \begin{matrix} a\alpha^2 + b\alpha + c = 0 \\ a\beta^2 + b\beta + c = 0 \end{matrix}$$

$$\rightarrow a(\alpha^n + \beta^n) + b(\alpha^{n-1} + \beta^{n-1}) + c(\alpha^{n-2} + \beta^{n-2})$$

$$a\alpha^n + b\alpha^{n-1} + c\alpha^{n-2} + a\beta^n + b\beta^{n-1} + c\beta^{n-2}$$

$$\alpha^{n-2}(a\cancel{\alpha^2} + b\cancel{\alpha} + c) + \beta^{n-2}(a\cancel{\beta^2} + b\cancel{\beta} + c)$$

$$\alpha^{n-2} \times 0 + \beta^{n-2} \times 0 = 0$$

Q Let α, β are Roots of $x^2 - 6x - 2 = 0$.

If $a_n = \alpha^n - \beta^n$; $n \geq 1$ then $\frac{a_{10} - 2a_8}{3a_9} = ?$

$$x^2 - 6x - 2 = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix} \Rightarrow \begin{matrix} \alpha^2 - 6\alpha - 2 = 0 & \times \alpha^8 \\ \beta^2 - 6\beta - 2 = 0 & \times \beta^8 \end{matrix}$$

$$\alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$$

$$\beta^{10} - 6\beta^9 - 2\beta^8 = 0$$

$$\begin{array}{r} \alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0 \\ \beta^{10} - 6\beta^9 - 2\beta^8 = 0 \\ \hline (\alpha^{10} - \beta^{10}) - 6(\alpha^9 - \beta^9) - 2(\alpha^8 - \beta^8) = 0 \end{array}$$

$$a_{10} - 6a_9 - 2a_8 = 0$$

$$a_{10} - 2a_8 = 6a_9$$

$$\left\{ \frac{a_{10} - 2a_8}{3a_9} \right\} = 2$$

Q Let α, β are 2 Real No. S.T

$\alpha + \beta = 1$ & $\alpha\beta = -1$ Let $P_n = \alpha^n + \beta^n$

$P_{n-1} = 11$ & $P_{n+1} = 29$ then $P_n^2 = ?$

$x^2 - (\alpha + \beta)x + \alpha\beta = 0 \leftarrow$ Q Eqn. Hodi hai

$$\boxed{x^2 - x - 1 = 0} \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$$

$$\alpha^2 - \alpha - 1 = 0 \times \alpha^{n-1}$$

$$\beta^2 - \beta - 1 = 0 \times \beta^{n-1}$$

$$\alpha^{n+1} - \alpha^n - \alpha^{n-1} = 0$$

$$+ \beta^{n+1} - \beta^n - \beta^{n-1} = 0$$

$$(\alpha^{n+1} + \beta^{n+1}) - (\alpha^n + \beta^n) - (\alpha^{n-1} + \beta^{n-1}) = 0$$

$$P_{n+1} - P_n - P_{n-1} = 0$$

$$29 - P_n - 11 = 0$$

$$P_n = 18$$

$$P_n^2 = 324$$

Q If α, β are roots of $5x^2 + 6x - 2 = 0$

If $S_n = \alpha^n + \beta^n$ then $5S_6 + 6S_5 = 2S_4$
[T/F]

$$5x^2 + 6x - 2 = 0 \xrightarrow{\alpha} \alpha \xrightarrow{\beta}$$

$$5\alpha^2 + 6\alpha - 2 = 0 \times \alpha^4$$

$$5\beta^2 + 6\beta - 2 = 0 \times \beta^4$$

$$5\alpha^6 + 6\alpha^5 - 2\alpha^4 = 0$$

$$\textcircled{+} \quad 5\beta^6 + 6\beta^5 - 2\beta^4 = 0$$

$$5(P_6) + 6P_5 - 2S_4 = 0$$

$$5P_6 + 6P_5 = 2S_4$$

Q Prod. of roots of $9x^2 - 18|x| + 5 = 0$?

$$9|x|^2 - 18|x| + 5 = 0$$

$$9|x|^2 - 15|x| - 3|x| + 5 = 0$$

$$3|x|(3|x| - 5) - 1(3|x| - 5) = 0$$

$$(3|x| - 5)(3|x| - 1) = 0$$

$$|x| = \frac{5}{3}, \frac{1}{3}$$

$$x = \frac{5}{3}, -\frac{5}{3}, \frac{1}{3}, -\frac{1}{3}$$

$$P_{\text{rod}} = \frac{25}{81}$$

Q. If α, β are roots of $x^2 - 64x + 256 = 0$

then $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}} = ?$

$$\alpha + \beta = 64, \alpha \cdot \beta = 256$$

$$\text{Demand} = \frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}}$$

$$\frac{64}{(2^8)^{5/8}} = \frac{64}{32} = 2$$

$$= \frac{\alpha^1 + \beta^1}{\alpha^{5/8} \cdot \beta^{5/8}} = \frac{64}{(\alpha\beta)^{5/8}} = \frac{64}{(256)^{5/8}}$$

Q. If α, β are roots of $x^2 + x \sin \theta - 2 \sin \theta = 0$

$\theta \in (0, \frac{\pi}{2})$ then $\frac{\alpha^{12} + \beta^{12}}{(\alpha^{-12} + \beta^{-12})(\alpha - \beta)^{24}}$

$$\alpha + \beta = -\sin \theta, \alpha \cdot \beta = -2 \sin \theta \quad \left| \begin{array}{l} \alpha^2 + \beta^2 \\ = (\alpha + \beta)^2 - 2\alpha\beta \end{array} \right.$$

$$\begin{aligned} \text{Demand} &= \frac{\alpha^{12} + \beta^{12}}{\left(\frac{\alpha^{12} + \beta^{12}}{(\alpha \cdot \beta)^{12}}\right)(\alpha - \beta)^{24}} \\ &= \frac{(\alpha \cdot \beta)^{12}}{(\alpha^2 + \beta^2 - 2\alpha\beta)^{12}} = \frac{(\alpha \cdot \beta)^{12}}{((\alpha + \beta)^2 - 4\alpha\beta)^{12}} \\ &= \frac{2^{12} \sin^{12} \theta}{(\sin^2 \theta + 8 \sin \theta)^{12}} = \frac{2^{12}}{(\sin \theta + 8)^{12}} \quad \text{A} \end{aligned}$$

Q10 If α, β are roots of $375x^2 - 25x - 2 = 0$

then $\lim_{n \rightarrow \infty} \sum_{r=1}^n \alpha^r + \lim_{n \rightarrow \infty} \sum_{r=1}^n \beta^r = ?$

$$\lim_{n \rightarrow \infty} \{ \alpha^1 + \alpha^2 + \alpha^3 + \dots + \alpha^n \} + \{ \beta^1 + \beta^2 + \beta^3 + \dots + \beta^n \}$$

$$\{ \alpha^1 + \alpha^2 + \alpha^3 + \dots \infty \} + \{ \beta^1 + \beta^2 + \beta^3 + \dots \infty \}$$

First term α
(.R. = α)

$$\frac{\alpha}{1-\alpha} \neq \frac{\beta}{1-\beta}$$

$$= \frac{(\alpha + \beta) - 2\alpha\beta}{(1-\alpha)(1-\beta)} = \frac{(\alpha + \beta) - 2\alpha\beta}{1 - (\alpha + \beta) + \alpha\beta}$$

$$= \frac{25}{375}$$

$$1 - \frac{25}{375} = 375$$

$$\sum_{r=1}^{\infty} \frac{a}{1-r}$$

$$Q_{11} \alpha = \text{Max}_{x \in \mathbb{R}} \left\{ 2^{2 \sin 3x} \cdot 4^{6 \cos 3x} \right\}$$

$$\beta = \text{Min}_{x \in \mathbb{R}} \left\{ 8^{2 \sin 3x} \cdot 4^{4 \cos 3x} \right\}$$

If $ax^2 + bx + c = 0$ in $\mathbb{Q}[x]$ whose roots are $\alpha^{\frac{1}{5}}, \beta^{\frac{1}{5}}$ then $(-b) = ?$

$$\alpha = \text{Max} \left\{ 2^{6 \sin 3x} \cdot 2^{8 \cos 3x} \right\} = \text{Max} \left\{ 2^{6 \sin 3x + 8 \cos 3x} \right\}$$

$$-\sqrt{6^2 + 8^2} \leq 6 \sin 3x + 8 \cos 3x \leq \sqrt{6^2 + 8^2}$$

$$-10 \leq \text{Min}_{x \in \mathbb{R}} (2^{-10}) \leq 2^{6 \sin 3x + 8 \cos 3x} \leq 2^{10} \rightarrow \text{Max.}$$

$$\alpha^{\frac{1}{5}} = (2^{10})^{\frac{1}{5}} = 4 \quad \beta^{\frac{1}{5}} = (2^{-10})^{\frac{1}{5}} = \frac{1}{4}$$

$$\alpha^{\frac{1}{5}} = 4, \beta^{\frac{1}{5}} = \frac{1}{4}$$

$$8x^2 + bx + c = 0$$

$$SOR = -\frac{b}{8} \mid POR = \frac{c}{8}$$

$$8(SOR) = -b \mid (= 8(POR))$$

$$(-b = 8 \{ POR + SOR \})$$

$$= 8 \left\{ 1 + 4 + \frac{1}{4} \right\} = 42$$

$$-\sqrt{a^2+b^2} \leq a \sin \theta + b \cos \theta \leq \sqrt{a^2+b^2}$$

$$-\sqrt{6^2+8^2} \leq 6 \sin 3x + 8 \cos 3x \leq \sqrt{6^2+8^2}$$

$$-10 \leq$$

$$\leq 10$$

$$2^{-10} \leq 2^1$$

$$\leq \underline{\underline{2^{10}}}$$